T-Comm "Телекоммуникации и транспорт", 2015, Vol. 9, №5, pp. 91 – 94.

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Models of incoming traffic in packet networks

Introduction

Development of the telephone networks required elaboration of teletraffic theory [1]. One of the main problems of this theory is investigation of the characteristics of incoming traffic. For analogue and digital telephone network, these characteristics have been well studied [2]. Obtained results are not always suitable for research of packet networks. New methods of investigation were based on stochastic fractals [3]. In particular, the heavy-tailed distributions [4] are used to describe the incoming traffic. Such approach yielded a number of useful results.

On the other hand, some teletraffic models may be analyzed by means of the distributions that are defined on interval of finite length. These distributions shall be denoted below as $A_{fl}(t)$. Subscript "fl" is the first letters in the words "finite length ". Such distributions do not have tails. Present article is devoted to the study of teletraffic models using distributions $A_{fl}(t)$.

Statement of the problem

Consider a sequence of requests on the input of the teletraffic system. This sequence is shown in Figure 1 on the axis "time". It consists of the eight requests. Duration between adjacent requests i and $i+1$ equals to t_i.

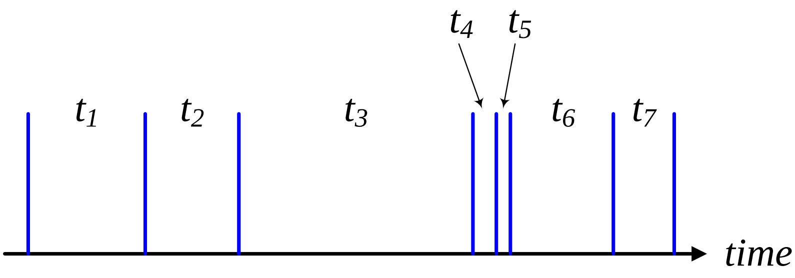

Figure 1. Incoming traffic as a sequence of requests

Value of t_i is random variable. It is well known [5] that all characteristics of random variable can be calculated from its distribution function $A(t)$. The main part of teletraffic models describing packet networks is based on distribution functions that are defined on interval $[a, \infty)$. It is often assumed that $a = 0$.

The usefulness of interval $[0, \infty)$ should be discussed in more detail. Two cases are interesting. In first case the requests are the calls generated by telephone set. In second case the requests are the IP-packets produced by personal computer.

In first case value t_i may indeed be either very small or very large. For the Plain Old Telephone Service [6], function $A(t)$ is well described by an exponential distribution. Sometimes this function is well represented with heavy-tailed distributions. Functions defined within interval $[0, \infty)$ are generally used for such types of distributions. Interval of the argument t has infinite length. Corresponding distributions shall be denoted below as $A_{il}(t)$. Subscript "*il*" is the first letters in the words "infinite length ".

For the second case, the situation is different. During each communication session, an exchange by significant amount of IP-packets takes place. Assume that first packet has appeared at the moment α. It means next packet will arrive up till time moment β with probability close to 1. The value $(\beta - \alpha)$ depends on some characteristics of packet network, types of traffic, etc. In any case, this value is relatively small. Therefore function $A_{fl}(t)$ should be selected from the distributions defined within interval $[\alpha, \beta]$.

Incoming traffic aggregated in the input of switching node depends on number of connected user-network interface and form of function. If this function can be presented by distribution $A_{il}(t)$ then incoming traffic corresponds to distribution $A(t)$ defined within interval $[0,\infty)$. Otherwise incoming traffic will be described by means of distribution $A(t)$ defined within interval of finite length.

Random variable t_i has a number of features. One of them is high value of coefficient of variation for some types of traffic. This fact is confirmed by the results of measurements. The considered task is investigation of difference between characteristics of requests delays for two queueing systems. Distinction between these systems lies in the type of functions $A(t)$. First function belongs to the type $A_{il}(t)$. Second function belongs to the type $A_{fl}(t)$. For both types of functions, average values $A^{(1)}$ and coefficients of variation C_A are assumed to be identical.

Considered models

Consider two teletraffic models belonging to the $G/D/1$ family. This designation is based on Kendall's notation [7]. Digit "1" in a third position means that the system uses a single service unit. The second symbol indicates that a service time of requests is constant. This assumption is appropriate for packet switches. Letter "G" in the first position is used for input process with general distribution law. Functions $A_{fl}(t)$ will define the first teletraffic models. For the second teletraffic models, functions $A_{il}(t)$ will be used.

Assume that both types of distributions have finite values of the mean $A^{(1)}$ and the coefficient of variation C_A. Both models are investigated for different ranges of C_A. For the first and second examples, $C_A \le 1$. Condition $C_A > 1$ is used for the third example.

First example

First model with function $A_{fl}(t)$ is presented by uniform distribution on the interval $[\alpha, \beta]$. For the same model, Erlang distribution with k as the shape parameter is used as example of function $A_{il}(t)$. Value λ is called the rate parameter for Erlang distribution. In Kendall's notation, studied models are referred to as $U/D/1$ and $E_k/D/1/$ respectively.

Distribution characteristics $A^{(1)}$ and C_A should be identical for the both functions. This condition allows us to write the following expressions [5]:

$$\frac{\alpha+\beta}{2}=\frac{1}{\lambda}, \quad \frac{\beta-\alpha}{(\beta+\alpha)\sqrt{3}}=\frac{1}{\sqrt{k}}. \tag{1}$$

The simplest solution is obtained when $k=3$. Then $\alpha=0$. In this case values β and λ are interconnected as follows:

$$\beta=\frac{2}{\lambda}. \tag{2}$$

Mean delay time $S^{(1)}$ and corresponding coefficient of variation C_S are considered as the functions of the load ρ. This value is calculated as the ratio of $B^{(1)}$ to $A^{(1)}$. Hereinafter, the value of $B^{(1)}$ is equal to one. For the proposed model, $B^{(1)}$ is mean value of processing time of requests.

Delay characteristics were obtained by simulation. Estimation of functions $S^{(1)}=F(\rho)$ indicate that substantial delay is inherent in the distributions $A_{fl}(t)$. The distinction of values $S^{(1)}$ is explained by the distribution of busy period [8] for both models. Functions $C_S=F(\rho)$ are shown in figure 2. Obviously the coefficient of variation C_S are also more for distributions $A_{fl}(t)$.

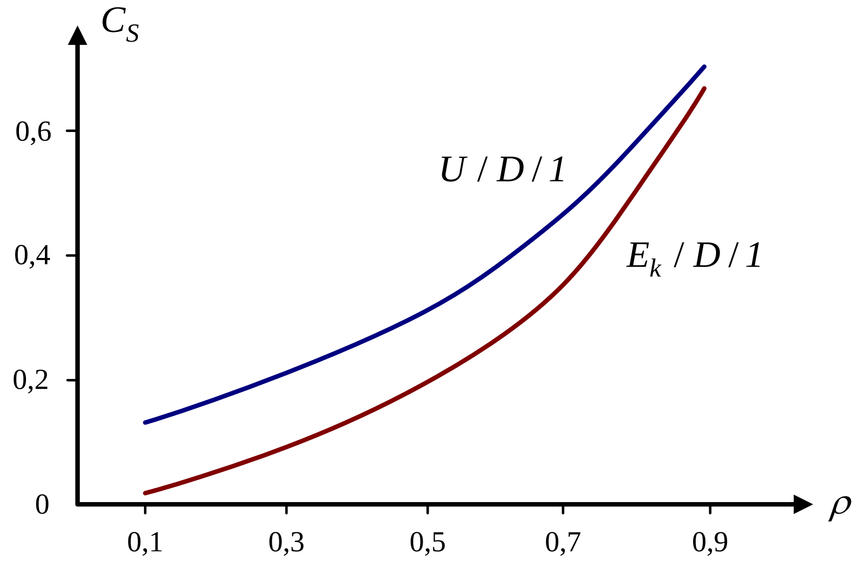

Figure 2. Coefficient of variation C_S as a function of load ρ under $C_A < 1$

It is interesting to check the validity of these conclusions under other distributions of service time. One of other distributions is considered in the next section of the article.

Second example

Similar results can be obtained for models $U/M/1$ and $E_k/M/1$. Symbol "M" corresponds to exponential distribution of service time. For both model, distribution function of the delay $S(t)$ is well known [8]:

$$S(t) = 1 - exp\left[-(1-\upsilon)\frac{t}{B^{(1)}}\right]. \tag{3}$$

Value υ lies within the range $(0,1)$. For calculation of this value, Laplace transform $A^*(s)$ of the function $A(t)$ should be found. Then υ will be root of the following equation:

$$\upsilon = A^*\left(B^{(1)} - \upsilon B^{(1)}\right). \tag{4}$$

Laplace transforms $A^*(s)$ for functions $A_{fl}(t)$ and $A_{ll}(t)$ are given for instance in handbook [9]. Values υ calculated for the second example are shown in table 1. The difference between the values υ for both models are placed in the bottom row of the table.

Table 1. Values υ for considered models and their difference

Load ρ	0,1	0,3	0,5	0,7	0,9
υ_1, model $E_k / M / 1$	0,013	0,132	0,331	0,575	0,842
υ_2, model $U / M / 1$	0,053	0,183	0,361	0,587	0,853
Difference, %	75,5%	27,9%	8,3%	2,0%	1,3%

For this example, $\upsilon_2 > \upsilon_1$. This means that average delay and its variance will always be greater for distributions of the form $A_{fl}(t)$. Calculations based on some other distributions of service time led to the following conclusion: for any distribution of service time two inequalities are true:

$$^{fl}S^{(1)} >^{il} S^{(1)}, \quad ^{fl}C_S >^{il} C_S. \tag{5}$$

Left superscript at the parameters of the delay determines the nature of the incoming traffic.

Third example

In some cases, coefficient of variation C_A is more than one. To solve this problem the following models are suitable: $B/D/1$ and $W/D/1$. Symbols "B" and "W" are used for designation of beta distribution and Weibull distribution respectively.

For beta distribution with parameters u and v, values $A^{(1)}$ and C_A are calculated by the following formulas [5]:

$$A^{(1)} = \frac{u}{u+v}, \quad C_A = \sqrt{\frac{v}{u(u+v+1)}}. \tag{6}$$

For investigation of the model $B/M/1$ it is needed to define parameters u and v. These values are estimated from the formulas (6):

$$v = \frac{\left[1 - A^{(1)}\right]\left[1 - A^{(1)}\left(1 + C_A^2\right)\right]}{A^{(1)} C_A^2}, \quad u = \frac{1 - A^{(1)}\left[1 + C_A^2\right]}{C_A^2}. \tag{7}$$

For Weibull distribution with parameters a and c, values $A^{(1)}$ and C_A are calculated by the following formulas [5]:

$$A^{(1)} = a\Gamma\left(\frac{1}{c}+1\right), \quad C_A = \sqrt{\frac{\Gamma\left(\frac{2}{c}+1\right)}{\Gamma^2\left(\frac{1}{c}+1\right)} - 1}. \tag{8}$$

Values $A^{(1)}$ and C_A calculated by formulas (6) and (8) should be identical. Parameters a and c from expressions (8) can be found only numerically.

Analysis of the functions $S^{(1)} = F(\rho)$ shows that a substantial delay is inherent in the distributions $A_{fl}(t)$. Thus the conclusion obtained for the first and second examples is valid also for large values of the coefficient of variation C_A. Behaviour of functions $S^{(1)} = F(\rho)$ are depicted in figure 3. These functions are plotted for coefficient of variation $C_A = 10$. Delay characteristics were obtained by simulation as in the first example.

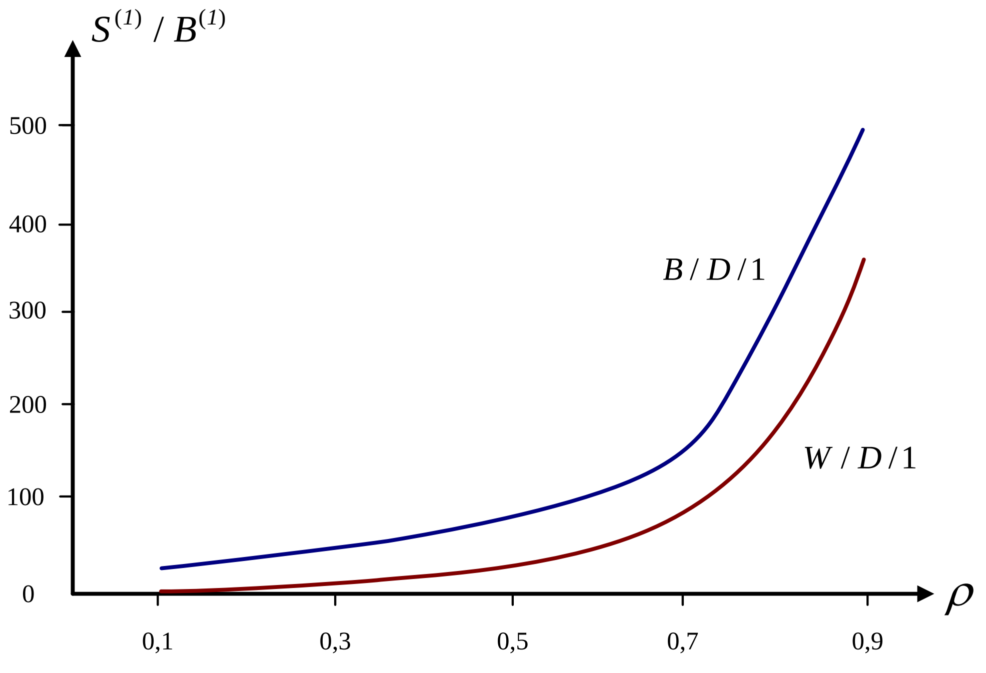

Figure 3. Average delay $S^{(1)}$ as a function of load ρ under $C_A > 1$

The values on the y-axis indicate the possibility of a large queue of demands. For such models, waiting time distribution becomes asymptotically exponential one [10]. This means $C_S \approx 1$.

The usage of other distributions does not change the character of the curves. In particular, this conclusion was confirmed using hyperexponential distribution and Pareto distribution. This means inequalities (5) are also true for the distributions of incoming traffic with high values of the coefficient of variation C_A.

Conclusion

The purpose of this article is simple. The distributions that are defined on interval of finite length are very useful for investigation of incoming traffic in packet networks. Analytical expressions and simulation results confirmed the effectiveness of the proposed approach to the description of the downstream applications.

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Abstract

A significant portion of the investigations related to the incoming traffic in packet networks is based on models with so-called heavy-tails. This article focuses on models that can be called "without tails". This name may be used due to the fact that we consider the distribution of incoming traffic defined on a finite time interval. Models with such distributions describe the incoming traffic in packet networks more correctly. Mean delay and coefficient of variation for proposed models are larger than the same values in the models with heavy-tails.

Keywords**:** *incoming traffic, teletraffic model, distribution, mean delay, coefficient of variation.*